Gravitational waves in modified teleparallel theories

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Introduction

- General relativity is a very successful theory of gravity ... BUT:
  - Small-scale problems: quantization of gravity
  - Large scale problems: accelerated expansions during inflation and the current late expansion
- One possible solution is to consider modifications/extensions of general relativity- at least as toy models
- In this talk:
  - Introduce teleparallel gravity theories
  - Show some recent results interesting for gravitational waves astronomy

Based on

M. Hohmann, M. Krššák, C. Pfeifer and U. Ualikhanova,
Propagation of gravitational waves in teleparallel gravity theories
Phys. Rev. D **98**, no. 12, 124004 (2018) [arXiv:1807.04580 [gr-qc]].

General relativity

General relativity is built on **two inputs:**

1. The underlying geometry is the **Riemannian geometry** given by the unique torsion-free and metric compatible connection

$$\overset{\circ}{\Gamma}{}^{\rho}{}_{\nu\mu} = \frac{1}{2} g^{\rho\sigma} \left(g_{\nu\sigma,\mu} + g_{\mu\sigma,\nu} - g_{\nu\mu,\sigma} \right)$$

resulting in geometry being attributed to the curvature tensor

$$R^{a}{}_{b\mu\nu} \neq 0$$

2. The Lagrangian is the **Einstein-Hilbert** one

$$\mathcal{L}_{EH} = \frac{\sqrt{-g}}{2\kappa} R$$

resulting in Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa \Theta_{\mu\nu}$$

Uniqueness of general relativity and its modifications

- Riemannian geometry is a unique geometry where connection is expressed in terms of the metric only
- EH Lagrangian is the unique one with second order field equations
  - Rather unexpected due to the presence of $\partial^2 g$

$$R(g, \partial g, \partial^2 g)$$

  - guaranteed by the **Lovelock's theorem**
- To modify GR we need to add more fields, break locality, or consider higher order theories and deal with ghosts:

- $f(R)$ gravity- an interesting exception– higher order theory equivalent to scalar-tensor gravity–without ghosts

Teleparallel theories of gravity

- The other way to change GR is to not change just the Lagrangian but also the geometry
- In differential geometry the spacetime (metric) and the rule of parallel transport (connection) are independent structures
- We can define different connections at the same manifold and use it to describe the same physics (Riemannian is just one choice of many)
- One particular choice is **teleparallel geometry**
- Vectors are being **"parallel at distance"** (Latin: tele-parallel)

History

- First time used in Einstein's (unsuccessful) attempt to unify gravity and EM theory in 1920s
- Revived in 1960s and 1970 by Moller, Cho, Shirafuji, Hayashi, ... as an alternative to GR (without unification)

Teleparallel geometry

- Tetrad formalism: fundamental variables are
  tetrad ($h^a{}_\mu$) and **spin connection** ($\omega^a{}_{b\mu}$)

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu . \qquad \mathcal{D}_\nu X^a = \partial_\nu X^a + \omega^a{}_{b\nu} X^b ,$$

- Define geometry by the **metric-compatibility** and **zero curvature** (teleparallel constraint)
- Solved by the pure-gauge **teleparallel spin connection**

$$\omega^a{}_{b\mu} = \Lambda^a{}_c \partial_\mu \Lambda_b{}^c , \qquad \Lambda_b{}^c = (\Lambda^{-1})^b{}_c$$

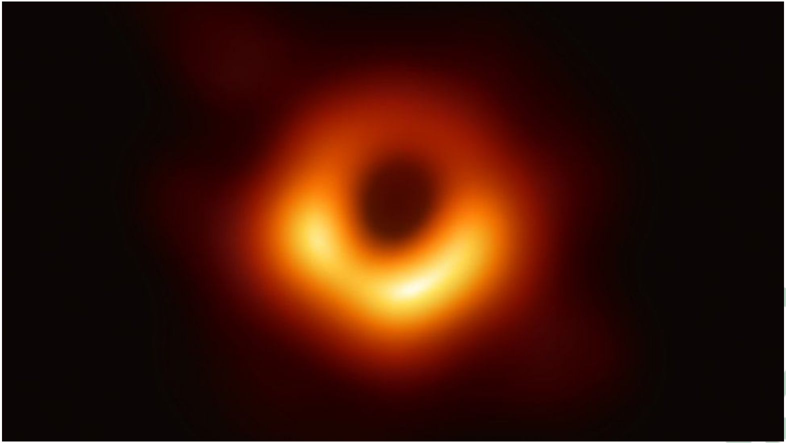
- or the **teleparallel affine connection**

$$\Gamma^\rho{}_{\nu\mu} = h_a{}^\rho \partial_\mu h^a{}_\nu + h_a{}^\rho \omega^a{}_{b\mu} h^b{}_\nu .$$

- The non-trivial geometry is (by definition) attributed to the **torsion** $T^\rho{}_{\mu\nu}$ of spacetime

Teleparallel geometry:example

- Torsion (by definition) describes non-trivial geometry of spacetime
- Recent example



Teleparallel formulation of GR: poor man's approach

1. Use the **Ricci theorem** of differential geometry relating $\Gamma^{\rho}{}_{\nu\mu}$ and $\overset{\circ}{\Gamma}{}^{\rho}{}_{\nu\mu}$

$$\Gamma^{\rho}{}_{\nu\mu} = \overset{\circ}{\Gamma}{}^{\rho}{}_{\nu\mu} + K^{\rho}{}_{\nu\mu}$$

2. and apply it to the Ricci scalar

$$R = -T + B,$$

where

- $T = T = \frac{1}{4} T^{a}{}_{\mu\nu} T_{a}{}^{\mu\nu} + \frac{1}{2} T^{a}{}_{\mu\nu} T^{\nu\mu}{}_{a} - T^{\nu\mu}{}_{\nu} T^{\nu}{}_{\mu\nu}$ is the torsion scalar
- $B = \frac{2}{h}\partial_{\mu}(h T^{\nu\mu}{}_{\nu})$ is the boundary term

Teleparallel formulation of GR: poor man's approach

3. and apply it to Einstein-Hilbert Lagrangian given by the Ricci scalar

$$\mathcal{L}_{EH} = \frac{h}{2\kappa} R$$

to show that

$$\mathcal{L}_{EH} = \mathcal{L}_{TG} + \partial_\mu \left(\frac{h}{\kappa} T^{\nu\mu}{}_{\nu} \right)$$

to show that

4. (Throw away the boundary term) and consider

Teleparallel Lagrangian

$$\mathcal{L}_{\text{TG}} = \frac{h}{2\kappa} T = \frac{h}{2\kappa} \left[\frac{1}{4} T^{a}{}_{\mu\nu} T_{a}{}^{\mu\nu} + \frac{1}{2} T^{a}{}_{\mu\nu} T^{\nu\mu}{}_{a} - T^{\mu} T_{\mu} \right]$$

Teleparallel formulation of GR

Teleparallel Lagrangian

$$\mathcal{L}_{\text{TG}} = \frac{h}{2\kappa} T = \frac{h}{2\kappa} \left[\frac{1}{4} T^{a}{}_{\mu\nu} T_{a}{}^{\mu\nu} + \frac{1}{2} T^{a}{}_{\mu\nu} T^{\nu\mu}{}_{a} - T^{\mu} T_{\mu} \right]$$

- is equivalent (up to a boundary term) to GR and hence defines **teleparallel equivalent of general relativity**
- is diffeomorphism and locally Lorentz invariant (!) invariant
- varying with respect to tetrad leads to

Teleparallel field equations

$$\mathcal{D}_{\sigma} \left(h S_{a}{}^{\rho\sigma} \right) - h \kappa t_{a}{}^{\rho} = \kappa h \Theta_{a}{}^{\rho},$$

Superpotential $S_a{}^{\mu\nu} = \frac{1}{2} \left(T^{\nu\mu}{}_{a} + T_{a}{}^{\mu\nu} - T^{\mu\nu}{}_{a} \right) - h_{a}{}^{\nu} T^{\sigma\mu}{}_{\sigma} + h_{a}{}^{\mu} T^{\sigma\nu}{}_{\sigma}.$

Gravitational energy-momentum tensor $t_a{}^{\rho} = \frac{1}{\kappa} h S_{b}{}^{\sigma\rho} T^{b}{}_{\sigma a} + h_{a}{}^{\rho} \mathcal{L}$

Matter energy-momentum tensor $\Theta_{a}{}^{\rho} = \frac{\delta \mathcal{L}_M}{\delta h^{a}{}_{\rho}}$

Why to consider teleparallel theories?

Teleparallel equivalent of general relativity is ”just” a different representation of general relativity and hence does not provide new solutions. However, it does offer:

1. Fundamental understanding of what is gravity: is gravity the curvature or torsion? Or both? Dual theory to general relativity!
2. Possibly a gauge theory, although rather unclear (see Morgan's talk)
3. New approach to problems of general relativity, e.g. definitions of energy-momentum, action regularization, etc...
4. **New modified theories of gravity (this talk)**

Modifications of teleparallel gravity: f(T) gravity

- Consider a generalization *a la* $f(R)$ gravity Ferraro and Fiorini 2007, Linder 2011

$$\mathcal{L}_{\mathsf{TG}} = \frac{h}{2\kappa} T \qquad \rightarrow \qquad \mathcal{L}_{\mathsf{f}} = \frac{h}{2\kappa} f(T)$$

- Since T contains only first derivatives, $f(T)$ has automatically second order field equations and hence **avoids Ostrogradsky ghosts**
- A very popular model with rich phenomenology and cosmological applications ($>$ 500 papers in 12 years)
- Initially thought to have problem with local Lorentz invariance but could be avoided if the theory is properly formulated Krssak and Saridakis 2016

See review:

Y. F. Cai, S. Capozziello, M. De Laurentis, E. N. Saridakis,
f(T) teleparallel gravity and cosmology,
Rept. Prog. Phys. **79** (2016) no.10, 106901 [arXiv:1511.07586 [gr-qc]].

Gravitational waves in f(T) gravity

- There are no extra modes Bamba, Capozziello, de Laurentis, Nojiri, Saez-Gomez 2013
- Interesting to understand why:
  - Perturbing the tetrad

$$h^a{}_\mu = \delta^a_\mu + \varepsilon \; u^a{}_\mu$$

  - the torsion scalar is proportional to

$$T \propto (\partial u)^2$$

  - unlike the Ricci scalar

$$R \propto \partial^2 u + (\partial u)^2$$

- An important insight into nature of degrees of freedom- non-perturbative and strong gravity problem
- Compared to GR the dispersion and frequency of GW is changed

Cai, Li, Saridakis, Xue, 2018

Towards most general modified teleparallel models

- $f(T)$ gravity is the simplest extension *a la* $f(R)$ gravity
- but there exists way more modified theories of gravity
- Two possible ways to generalize TG:
  - **Include higher derivatives:** $f(T, B)$ gravity Bahamonde, Böhmer and Wright 2015
  - **Or modify the torsion scalar**
    - New general relativity Hayashi, Shirafuji, 1979
    - $f(T_{\rm ax}, T_{\rm ten}, T_{\rm vec})$ Gravity Bahamonde, Böhmer, Krššák, 2017
    - premetric models Y. Itin, F. Hehl, Y. Obukhov 2017

Non-linear axiomatic models

M. Hohmann, L. Järv, M. Krššák and C. Pfeifer,
Teleparallel theories of gravity as analogue of nonlinear electrodynamics
Phys. Rev. D **97**, no. 10, 104042 (2018) [arXiv:1711.09930 [gr-qc]].

- At the level of perturbations the most general model with second order field equations is **New general relativity (NGR)**

$$\mathcal{L}_{\rm NGR} = \frac{h}{2\kappa} T_{\rm NGR} = \frac{h}{2\kappa}\left[c_1 T^a{}_{\mu\nu} T_a{}^{\mu\nu} + c_2 T^a{}_{\mu\nu} T^{\nu\mu}{}_a + c_3 T^\mu T_\mu\right]$$

Gravitational waves in NGR: analysis

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$$\mathcal{L}_{\text{NGR}} = \frac{h}{2\kappa} T_{\text{NGR}} = \frac{h}{2\kappa}\left[c_1 T^a{}_{\mu\nu} T_a{}^{\mu\nu} + c_2 T^a{}_{\mu\nu} T^{\nu\mu}{}_a + c_3 T^\mu T_\mu\right]$$

- Introduce perturbations

$$h^a{}_\mu = \delta^a_\mu + \varepsilon\ u^a{}_\mu$$

- split into symmetric (metric) and antisymmetric perturbations

$$u_{\mu\nu} = s_{\mu\nu} + a_{\mu\nu}$$

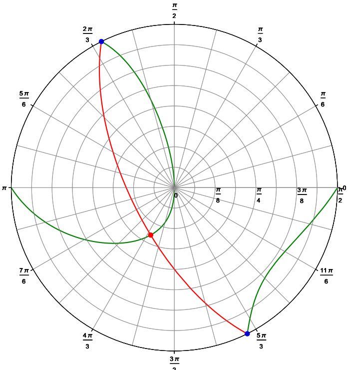
- Field equations at the perturbative order

$$\begin{aligned} 0 = \partial_\rho\big[&(2c_1 - c_2)\partial^\rho a^{\tau\kappa} - (2c_1 - c_2)\partial^\kappa a^{\tau\rho} + (2c_2 + c_3)\partial^\tau a^{\rho\kappa}\big] \\ &+ \partial_\rho\big[(2c_1 + c_2)\partial^\rho s^{\tau\kappa} - (2c_1 + c_2)\partial^\kappa s^{\tau\rho} + \big] \\ &\partial_\rho\big[c_3\big(\eta^{\tau\kappa}(\partial^\rho s^\beta{}_\beta - \partial_\beta s^{\rho\beta}) - \eta^{\tau\rho}(\partial^\kappa s^\beta{}_\beta - \partial_\beta s^{\kappa\beta})\big)\big]. \end{aligned}$$

Gravitational waves in NGR: results

M. Hohmann, M. Krssak, Ch. Pfeifer, U. Ualikhnanova, 2018

- All gravitational modes propagate with speed of light
- Number of possible polarizations:
  - **6:** $2c_1 + c_2 = c_3 = 0$ 2 blue points
  - **5:** $2c_1(c_2 + c_3) + c_2^2 = 0$ and $2c_1 + c_2 + c_3 \neq 0)$ green line
  - **3:** $2c_1(c_2 + c_3) + c_2^2 \neq 0$ and $2c_1 + c_2 + c_3 \neq 0$ white space
  - **2:** $2c_1 + c_2 + c_3 = 0$ and $c_3 \neq 0$ red line



Summary

- Different geometries can be used to describe gravity; not only the Riemannian one
- One particular (and probably the best motivated) choice is the teleparallel geometry
- Teleparallel theories of gravity are a natural framework to **both**
  1. Understand general relativity from a novel perspective
  2. Formulate and study new extended models of gravity
- For gravitational waves, to study the most general model with second order field it is sufficient to consider New General Relativity
- All GW modes propagate with speed of light and we have possibly 2,3,5,6 polarizations (depending on parameters)

For more details see

M. Hohmann, M. Krššák, C. Pfeifer and U. Ualikhanova,
Propagation of gravitational waves in teleparallel gravity theories
Phys. Rev. D **98**, no. 12, 124004 (2018) [arXiv:1807.04580 [gr-qc]].

谢谢!

Recent review:

M. Krššák, R. J. v. d. Hoogen, J. G. Pereira, C. G. Böhmer, A. A. Coley, *Teleparallel Theories of Gravity: Illuminating a Fully Invariant Approach*, ArXiV:1810.12932 [gr-qc].

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